Effects and Coeffects in Call-By-Push-Value

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```
1 | let x = "hello" in 2 | print "world"
```

```
1 | let x = "hello" in 2 | print "world"
```

can the first line be removed?

```
1 | let x = print "hello" in 2 | print "world"
```

is the code bound to x pure?

is the code bound to x pure?

is x unused in the code that follows?

x: string, y: nat \vdash print x: unit

 $x : string, y : nat \vdash print x : unit$

$$\Gamma \vdash e : \overset{\phi}{\tau}$$

IO effects

$$\Gamma \vdash e : \overset{\phi}{\tau}$$

IO effects

Running time

$$\Gamma \vdash e : \overset{\phi}{\tau}$$

IO effects

Running time

$$\Gamma \vdash e : \overset{\phi}{\tau}$$

Exceptions

x: string, y: nat \vdash print x: unit

x: string, y: nat \vdash print x: unit

$$x_1 : {}^{q_1}\tau_1, x_2 : {}^{q_2}\tau_2 \vdash e : \tau$$

$$\gamma \cdot \Gamma \vdash e : \tau$$

Relevance

$$\gamma \cdot \Gamma \vdash e : \tau$$

Relevance

Resources

$$\gamma \cdot \Gamma \vdash e : \tau$$

Relevance

$$\gamma \cdot \Gamma \vdash e : \tau$$

Differential privacy

Relevance

$$\gamma \cdot \Gamma \vdash e : \tau$$

Differential privacy

Information flow

Effects and Coeffects

Effect: is the code bound to x pure?

Coeffect: is x unused in the code that follows?

```
1 | let x = print "hello" in 2 | x; x
```

```
1 | let x = print "hello" in 2 | x; x
```

$$\rightarrow$$
 1 | let $x = print$ "hello" in 2 | $x; x$

$$\rightarrow 1 \mid let x = () in$$

$$2 \mid x; x$$

Expected effect?

hello

$$x \mapsto ()$$

 $1 \mid \text{let } x = \text{print "hello" in}$
 $\rightarrow 2 \mid x; x$

Expected effect?

hello

$$x \mapsto ()$$

$$1 \mid \text{let } x = \text{print "hello" in}$$

$$2 \mid (); ()$$

Expected effect?

hello

```
1 | let x = print "hello" in 2 | x; x
```

 \rightarrow 1 | let x = print "hello" in 2 | x; x

 $x \mapsto \text{print "hello"}$ $1 \mid \text{let } x = \text{print "hello" in}$ $\rightarrow 2 \mid x; x$

```
x \mapsto \text{print "hello"}

1 \mid \text{let } x = \text{print "hello" in}

\rightarrow 2 \mid \text{print "hello"; print "hello"}
```

```
x \mapsto \text{print "hello"}

1 \mid \text{let } x = \text{print "hello" in}

\rightarrow 2 \mid (); ()
```

Expected effect?

hello hello

Intermediate representation

Intermediate representation

Explicit evaluation order

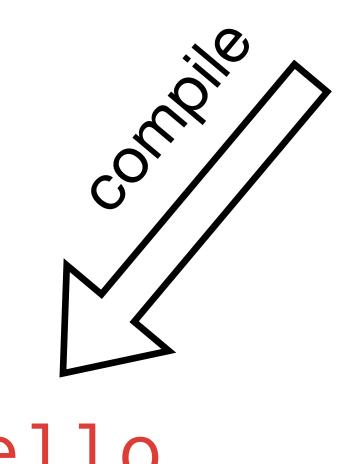
A subsuming paradigm

A subsuming paradigm

```
1 | let x = print "hello" in 2 | x;x
```

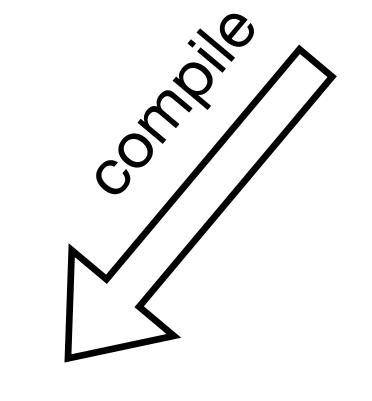
A subsuming paradigm

```
1 | let x = print "hello" in 2 | x;x
```

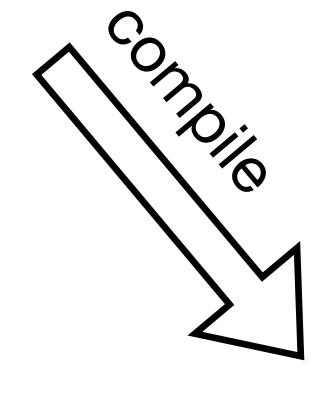


A subsuming paradigm

1 | let
$$x = print$$
 "hello" in 2 | $x;x$







hello hello

... in Call-By-Push-Value (CBPV) A subsuming paradigm

Expression

A subsuming paradigm

Value

Computation

A subsuming paradigm

Value

"is"

Computation

A subsuming paradigm

Value

"is"

A string is a value

Computation

A subsuming paradigm

Value

"is"

A string is a value

Computation

"does"

A subsuming paradigm

Value

"is"

A string is a value

Computation

"does"

Printing a string is a computation

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in } \\ 2 & x; x \end{vmatrix}$ Compilation 1

1 | let x = print "hello" in Compilation 1 2 | x; x | hello

 $1 \mid x \leftarrow \text{print "hello"}$

 $1 \mid x \leftarrow \text{print "hello"}$

 $\frac{1}{2} | \text{let } x = \text{print "hello" in } \text{Compilation 1}$ $\frac{1}{2} | x; x$

 $1 \mid x \leftarrow \text{print "hello"}$

Hprint "hello": F unit

 $1 \mid x \leftarrow \text{print "hello"}$ Hello": Funit "hello":

 $1 \mid x \leftarrow \text{print "hello"}$

x: unit

Hprint "hello": Funit

 $1 \mid x \leftarrow \text{print "hello"}$

 $x: unit \vdash N: B$ hprint "hello": Funit

 $\vdash x \leftarrow \text{print "hello"}; N:B$

 $1 \mid x \leftarrow \text{print "hello"}$

Hprint "hello": Funit

x: unit H N: B

 $\vdash x \leftarrow \text{print "hello"}(N)B$

 $1 \mid x \leftarrow \text{print "hello"}$

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in } \\ 2 & \text{kello} \end{vmatrix}$ Compilation 1

 $\begin{vmatrix} 1 \mid \text{let } x = \text{print "hello" in Compilation 1} \\ 2 \mid x; x \end{vmatrix}$ Compilation 1

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in } \\ 2 & \text{kello} \end{vmatrix}$ Compilation 1

x: unit

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in } \\ 2 & \text{kello} \end{vmatrix}$ Compilation 1

```
1 \mid x \leftarrow print "hello"
2 \mid \underline{\quad} \leftarrow \text{return } x
3 | return x
                   x: unit
```

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in } \\ 2 & x; x \end{vmatrix}$ Compilation 1

```
1 \mid x \leftarrow print "hello"
2 \mid \underline{\quad} \leftarrow \text{return } x
31 return x
```

 $\vdash x$: unit

Heturn x: Funit

```
1 | let x = print "hello" in Compilation 1 2 | x; x
```

```
1 \mid x \leftarrow print "hello"
2 \mid \underline{\quad} \leftarrow \text{return } x
31 return x
```

```
1 | let x = print "hello" in Compilation 1 2 | x; x
```

```
1 \mid x \leftarrow print "hello"
  2 \mid \underline{\quad} \leftarrow \text{return } x
  3 return x
```

```
let x = print "hello" in Compilation 1 x; x
```

```
1 \mid x \leftarrow return()
  2 \mid \underline{\quad} \leftarrow \text{return } x
  31 return x
```

```
1 | let x = print "hello" in Compilation 1
```

```
\chi \mapsto ()
   1 \mid x \leftarrow \text{print "hello"}
\rightarrow 2 \__ \tag{return } x
   31 return x
```

```
\chi \mapsto ()
   1 \mid x \leftarrow \text{print "hello"}
→ 2 | __ ← return ()
   3 return x
```

```
1 | let x = print "hello" in Compilation 1 2 | x; x
```

```
\chi \mapsto ()
      1 \mid x \leftarrow \text{print "hello"}
      2 \mid \underline{\quad} \leftarrow \text{return } x
\rightarrow 3 return x
```

```
1 | let x = print "hello" in Compilation 1 2 | x; x
```

```
\chi \mapsto ()
         1 \mid x \leftarrow \text{print "hello"}
         2 \mid \underline{\quad} \leftarrow \text{return } x
--- 31 return ()
```

```
1 | let x = print "hello" in Compilation 1 2 | x; x
```

```
1 \mid x \leftarrow \text{print "hello"}
2 \mid \underline{\quad} \leftarrow \text{return } x
3 return x
```

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in Compilation 2} \\ 2 & x; x \end{vmatrix}$

```
1 let x = print "hello" in Compilation 2 hello hello
```

 $1 \mid x \leftarrow \text{print "hello"}$

```
1 \mid x \leftarrow \{print "hello"\}
```

```
1 \mid x \leftarrow \{\text{print "hello"}\}
```

1 | let x = print "hello" in Compilation 2 hello hello

```
1 \mid x \leftarrow \{ print "hello" \}
```

Hprint "hello": F unit

1 | let x = print "hello" in Compilation 2 hello hello

```
1 \mid x \leftarrow \{print "hello"\}
```

Hprint "hello": Funit

 \vdash {print "hello"} : U(F)

1 | let x = print "hello" in Compilation 2 hello hello

```
1 \mid x \leftarrow \{print "hello"\}
```

 \vdash {print "hello"} : \mathbf{U} (\mathbf{F} unit)

```
1 \mid x \leftarrow return \{print "hello"\}
```

```
1 \mid x \leftarrow return \{print "hello"\}
```

```
1| x \leftarrow \text{return {print "hello"}}
2| x \leftarrow x
3| x \leftarrow x
```

```
1| x \leftarrow \text{return {print "hello"}}
2| x \leftarrow x
3| x \leftarrow x
                 x: U (F unit)
```

 $\begin{vmatrix} 1 & \text{let } x = \text{print "hello" in } \begin{vmatrix} \text{Compilation 2} \\ 2 & x; x \end{vmatrix}$

```
1 | x \leftarrow \text{return {print "hello"}}
2 | x \leftarrow x
3 | x \leftarrow x
```

 $x: \mathbf{U}$ (\mathbf{F} unit)

x: U (F unit)

x: U (F unit)

```
1| x \leftarrow \text{return {print "hello"}}
2| x \leftarrow x!
3| x!
```

-x: U (F unit)

-x!: Funit

```
1| x \leftarrow \text{return {print "hello"}}
2| x \leftarrow x!
3| x!
```

```
1 | let x = print "hello" in Compilation 2 | x; x
```

```
1 | let x = print "hello" in Compilation 2 | x; x
```

```
x \mapsto \{ print "hello" \}
      1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
\sim 2
      3 | x
```

```
1 | let x = print "hello" in Compilation 2 hello hello
```

```
x \mapsto \{\text{print "hello"}\}
1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}\}
2 \mid \underline{\quad} \leftarrow \{\text{print "hello"}\}\}
3 \mid \underline{\quad} x \mid
```

```
1 | let x = print "hello" in Compilation 2 | hello hello
```

```
x \mapsto \{\text{print "hello"}\}
1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
2 \mid \underline{\quad} \leftarrow \text{print "hello"}
3 \mid x \mid
```

```
1 | let x = print "hello" in Compilation 2 | hello
```

```
x \mapsto \{ print "hello" \}
       1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
\rightarrow 2 \leftarrow return ()
       3 | x
```

```
1 | let x = print "hello" in Compilation 2 | x; x
```

```
x \mapsto \{ print "hello" \}
1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
2 \leftarrow x
```

```
1 | let x = print "hello" in Compilation 2 | x; x
```

```
x \mapsto \{ print "hello" \}
     1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
```

```
1 | let x = print "hello" in Compilation 2 | x; x
```

```
x \mapsto \{ print "hello" \}
      1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
```

```
1 | let x = print "hello" in Compilation 2 | hello
```

```
x \mapsto \{ print "hello" \}
      1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
      --- 3 return ()
```

hello hello

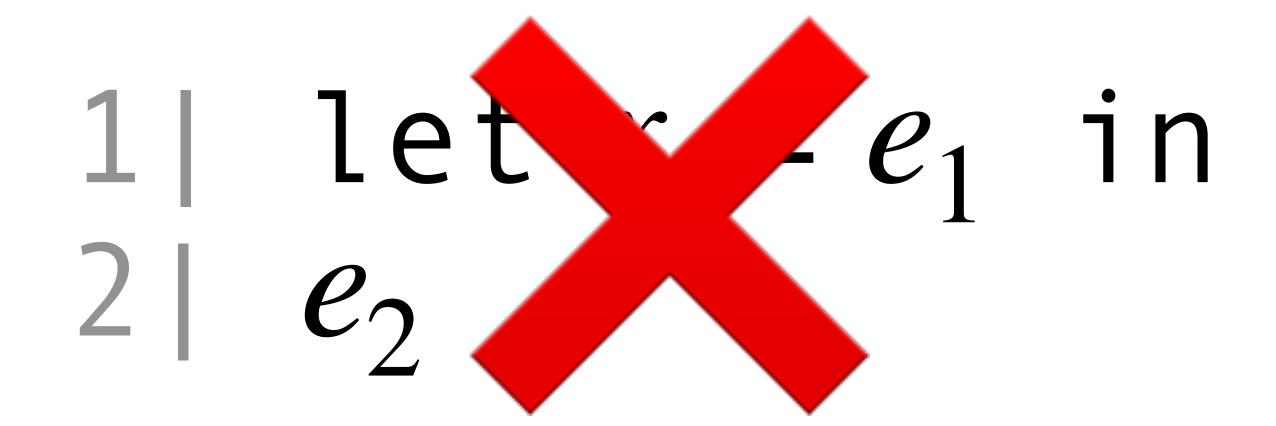
```
1 | let x = print "hello" in Compilation 2 | x; x
```

```
1 \mid x \leftarrow \text{return } \{\text{print "hello"}\}
2 \leftarrow x
3 \mid x!
```

hello hello

Dead code elimination

Dead code elimination



... in Call-By-Push-Value

... in Call-By-Push-Value

$$\Gamma \vdash M : FA$$

$$\Gamma, x: A \vdash N: B$$

Effects ... in Call-By-Push-Value

$$\Gamma \vdash M : {}^{\varepsilon} F A$$

 $\Gamma, x: A \vdash N: B$

Effect: is the code bound to x (i.e. M) pure?

Effects ... in Call-By-Push-Value

$$\Gamma \vdash M : {}^{\varepsilon} F A$$

$$\Gamma, x: A \vdash N:^{\phi}B$$

Effect: is the code bound to x (i.e. M) pure?

Effects and Coeffects in Call-By-Push-Value

$$\gamma_1 \cdot \Gamma \vdash M : ^{\varepsilon} F A$$

$$\gamma_2 \cdot \Gamma, x : A \vdash N : B$$

Effect: is the code bound to x (i.e. M) pure?

Coeffect: is *x* unused in *N*?

Effects and Coeffects in Call-By-Push-Value

$$\gamma_1 \cdot \Gamma \vdash M : {}^{\varepsilon} F A$$

$$\gamma_2 \cdot \Gamma, x : A \vdash N : B$$

Effect: is the code bound to x (i.e. M) pure?

Coeffect: is x unused in N?

Effects and Coeffects in Call-By-Push-Value

$$\gamma_1 \cdot \Gamma \vdash M : {}^{\varepsilon} F A$$

$$\gamma_2 \cdot \Gamma, x : A \vdash N : B$$

$$\gamma_1 \cdot \Gamma \vdash M : {}^{\varepsilon} F A$$

$$\gamma_2 \cdot \Gamma, x : A \vdash N : B$$

$$\gamma_1 \cdot \Gamma \vdash M : {}^{\varepsilon} \mathbf{F} A$$
 $\gamma_2 \cdot \Gamma, x : {}^{\circ} A \vdash N : {}^{\phi} B$

$$\gamma_2 \cdot \Gamma \vdash x \leftarrow_{\varepsilon}^{\mathbf{0}} M ; N :^{\phi} B$$

demand

$$\gamma_1 \cdot \Gamma \vdash M : {}^{\varepsilon} F A$$

$$\gamma_2 \cdot \Gamma, x : A \vdash N : B$$

$$\gamma_2 \cdot \Gamma \vdash x \leftarrow_{\varepsilon}^{0} M ; N :^{\varphi} B$$



$$X \leftarrow_{\varepsilon}^{0} M \equiv N$$

$$\frac{x}{N} \leftarrow_{\varepsilon}^{0} M = N$$



$$\rho \vdash M \Downarrow T$$

 $\rho \vdash M \Downarrow T$

Semantics

$$_{environment} \vdash M \Downarrow T^{kerminal}$$

Effect-tracking semantics

$$\rho \vdash M \Downarrow T \# \phi$$

Effect-tracking semantics

$$\rho \vdash M \Downarrow T \# \phi$$

resulting

Effect-tracking and coeffect-tracking semantics

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$

resulting

Effect-tracking and coeffect-tracking semantics

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$



Effect-tracking and coeffect-tracking semantics

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$

$$\Gamma \vdash M: B$$

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$

$$\Gamma \vdash M : B$$

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$

Type-and-effect soundness

$$\Gamma \vdash M : B$$

$$\phi \leq \phi'$$

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$



Type-and-effect soundness

$$\gamma' \cdot \Gamma \vdash M : ^{\phi'}B$$

$$\phi \leq \phi'$$

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$



Type-and-effect-and-coeffect soundness

$$\gamma \leq \gamma'$$
 $\gamma' \cdot \Gamma \vdash M : B$

$$\phi \leq \phi'$$

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$



 $\Gamma \vdash M : B$

 $\Gamma \vdash \{M\} : UB$

 $\Gamma \vdash V : A$

 Γ Hreturn $V: \mathbf{F} A$

 $\Gamma \vdash M : B$

 $\Gamma \vdash \{M\} : U_{\phi}B$

 $\Gamma \vdash V : A$

 Γ Hreturn $V:\mathbf{F}A$

 $\Gamma \vdash M : B$

 $\Gamma \vdash \{M\} : U_{\phi}B$

suspended

 $\Gamma \vdash V : A$

 Γ Hreturn $V:\mathbf{F}A$

$$\gamma \cdot \Gamma \vdash M : B$$

$$\gamma \cdot \Gamma \vdash \{M\} : \mathbf{U}_{\phi}B$$
suspended
effect

$$\gamma \cdot \Gamma \vdash V : A$$

 $(q \cdot \gamma) \cdot \Gamma \vdash \text{return}_q V : \mathbf{F}_q A$

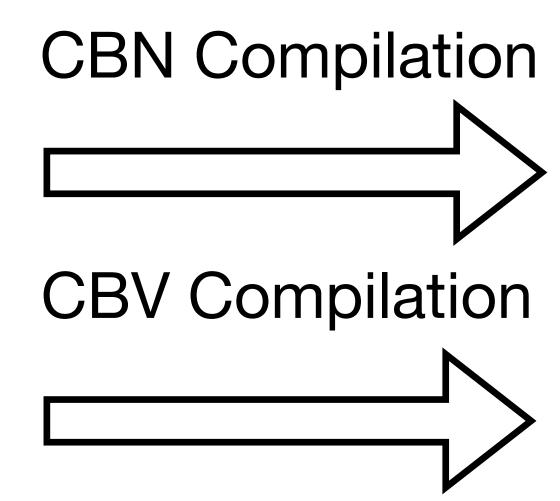
$$\gamma \cdot \Gamma \vdash M : B$$

$$\gamma \cdot \Gamma \vdash \{M\} : \mathbf{U}_{\phi}B$$
suspended
effect

$$\gamma \cdot \Gamma \vdash V : A$$
 $(q \cdot \gamma) \cdot \Gamma \vdash \operatorname{return}_q V : \mathbf{F}_q A$

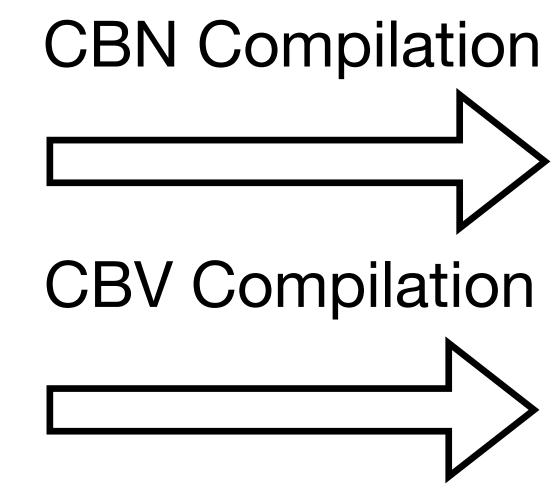
coeffect





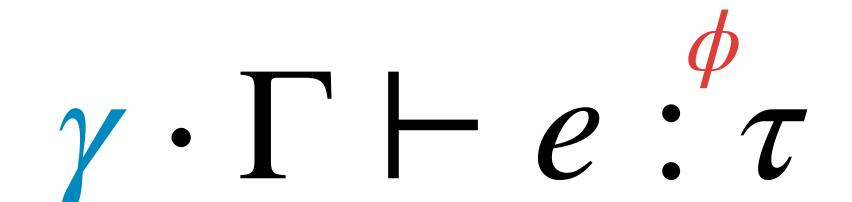
Call-By-Push-Value

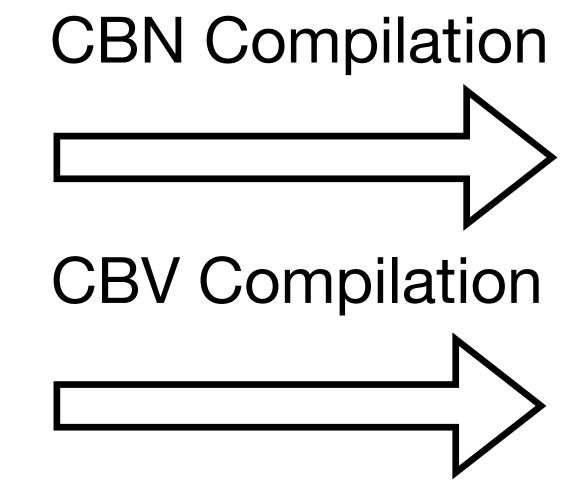


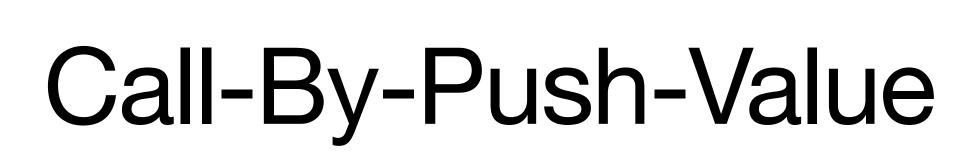


Call-By-Push-Value









preserve coeffects preserve



$$\gamma \cdot \Gamma \vdash M : B$$

Thank you!

$$\gamma \cdot \rho \vdash M \Downarrow T \# \phi$$

$$x \leftarrow_{\varepsilon}^{o} M; N \equiv N$$



mechanized in

